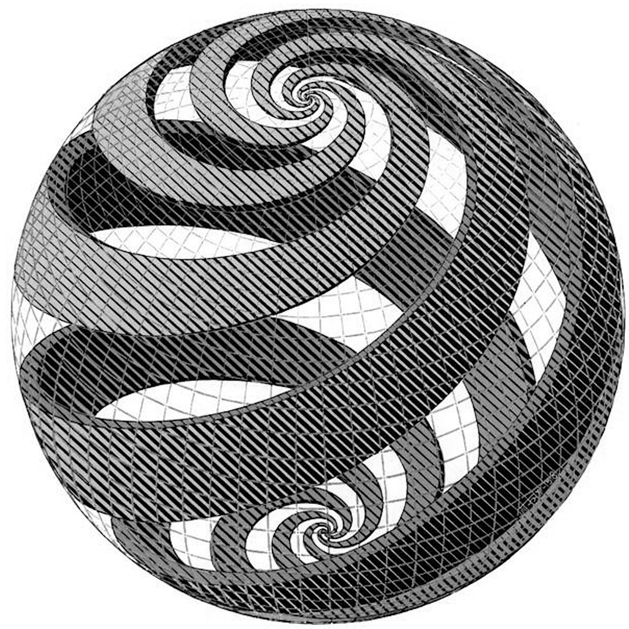
Trigate

a quantum cryptocurrency project

Initial position paper
by: Rev Jonathan Barlow Gee
July 30, 2024.

Contents::

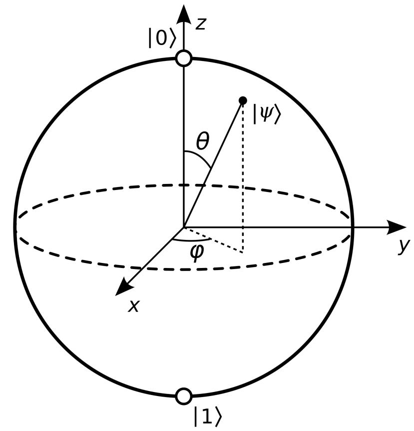


Abstract:
Three phases are identified toward establishing useable quantum cryptocurrency. The first phase establishes a trinary quantum switch-gate as superior to a standard qubit for use in quantum cryptocurrency coding languages by using the trinary gate to construct a tetra-helix blockchain as a radius in a spherical encoding network. The second phase explores the methods of projecting a Euclidian plane space onto the surface of a sphere in order for the tetra-helix radius to be translated into complexity on a relative sphere's surface area. The third phase proposes an application capable of storing quantum data as sums of spin inside a perfect magic number hypercube to balance a "wallet" or "portfolio" for housing multiple accounts with differing parameter quantum cryptocurrencies. Following this further implications maybe explored.

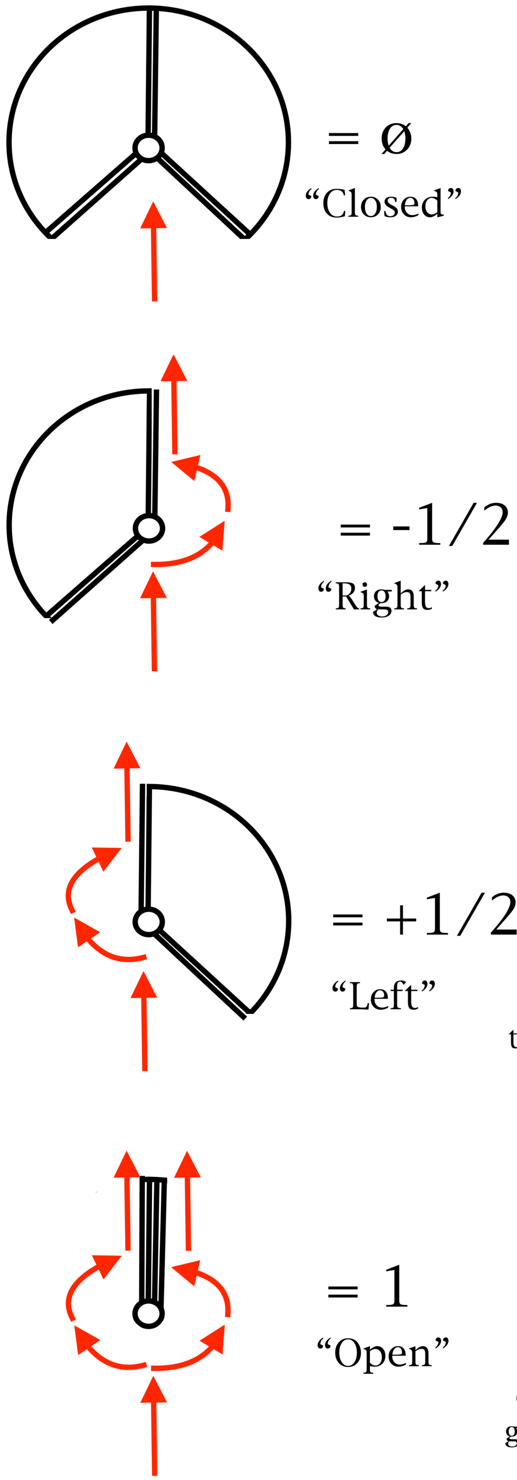
Key Terms:
https://en.wikipedia.org/wiki/Cryptocurrency
https://en.wikipedia.org/wiki/Ternary_computer
https://en.wikipedia.org/wiki/Boerdijk–Coxeter_helix
https://en.wikipedia.org/wiki/Riemann_sphere
https://en.wikipedia.org/wiki/Bloch_sphere
https://en.wikipedia.org/wiki/Magic_hypercube

Phase 1:

Qubit or Bloch Sphere as Binary



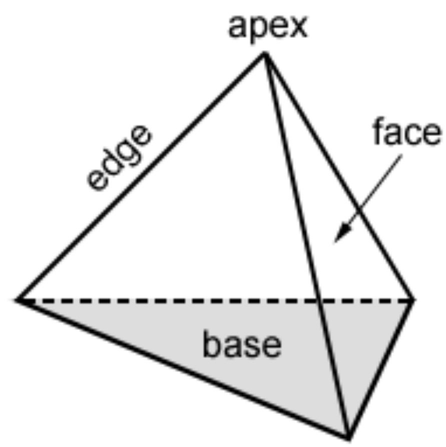
The Bloch sphere measures the probability of a figurative "vector" on a mathematical construct depicting the space between "ø" and "1" - where "|ø>" symbolizes an absolute "negative" or "closed state" and where "|1>" symbolizes an absolute "positive" or "open state." The location of the figurative "vector" on this spherical mathematical construct relative to either of these poles determines its probability or likelihood of maintaining the same "vector" uninterrupted. Because modern scientific methods of testing quantum states directly involve interrupting their vector - by striking an electron cloud with a photon to collapse both into a singular electron charge for the duration of one Planck time, for example - they usually succeed at measuring only either the direction of trajectory or the rate of velocity of the quantum they disturb - for example, by comparing the reflected trajectory or angle of incidence of the photon used to collapse the electron cloud to the observed position and location of the temporary electron charge. The mathematical construct of the "Bloch sphere" circumvents this shortcoming of modern quantum physics by providing a construct for modeling any figurative "vector" as a combination of the trajectory (called θ) and the velocity (called φ), thus allowing one to plot a relative physical location for the measured quantum - that includes both its projected future trajectory and its rate of velocity into one, figurative "vector" (called ψ) - mapped onto the surface of the mathematical construct of the Bloch sphere itself. If the figurative vector (ψ) appears closer to the |ø> "negative" probability, then the data about this quantum position on the Bloch sphere should be considered "unreliable;" if the vector (ψ) appears closer to the |1> "positive" probability, then the data about this quantum position on the Bloch sphere should be considered "reliable." In short, "ø" equals "false" and "1" equals "true" in all standard binary computer processing code languages. If the binary code reads "00100000," for example, this computes as a sequence of "true" or "false" (or probabilistically "more likely true" or "more likely false"), where "true" (or 1) symbolizes an "open switch gate" and "false" (or ø) symbolizes a "closed switch gate." Thus, a computer reading this code translates it as the sequence of two "closed" gates followed by one "open" gate, followed by 5 "closed" gates and interprets this to mean the "space" between two words in a sentence. If we want to apply this code to translating a message (comprised of words) we can substitute sets of such "1's and ø's" of any sum (usually 8, 10 or 16 are used) per letter in the text message. For example, if we see "01101000 01100101 01101100 01101100 01101111 00100000 01110100 01101000 01100101 01110010 01100101" a computer programmed to understand oct-binary code language will see, "hello there". This method is essentially similar to "Morse code," where sequences of "dots" and "dashes" combine to form letters spelling out words in a long string of coded language. The mathematical construct of the Bloch sphere thus measures basic "binary" in the quantum scale of pure probabilities by its scalar axis between the opposite poles labeled "|ø>" and "|1>", but with an added layer of encryption provided by the "uncertainty" of the figurative vector.



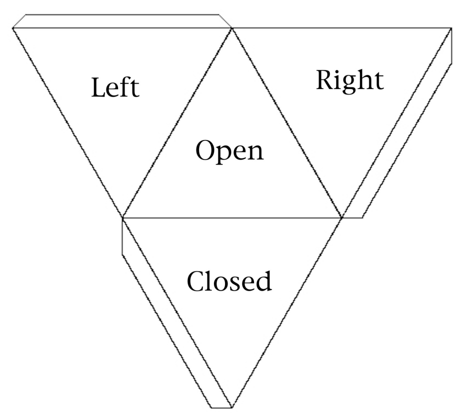
Trinary Quantum Switch Gate

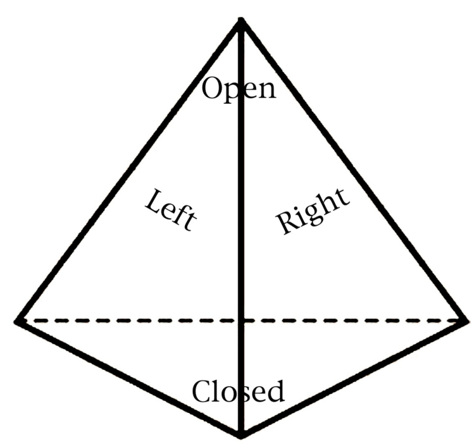
In “binary,” a switch gate can only be either “open” or “closed.” In “trinary,” a switch gate can have 3 positions besides “Closed.” These positions are “Open” (which is opposite of “closed”), “Right” (which is -1/2) and “Left” (which is +1/2). When a pulsed signal enters a trinary switch gate, the signal can be paused (“closed”), spun (“right” or “left”), or be allowed to transmit double its pulse rate (when the circuit is fully “open”). The switch gate is usually “closed” or “open” only 1/2 on the “right” or “left,” and it is only fully “open” after a certain, random number of cycles. Therefore, the quantum switch gate introduces uncertainty in two phases: 1. whether the “closed” gate will open “right” or “left,” and 2. when the gate will fully “open.” While an electrical signal would tend to lose charge over time in such a system (where the gate is only fully “open” at most 1/4th of the time), a “Qubit” (as a pulsed signal of electrons entangled to a photon) can exploit the quantum nature of light and pass through either gate (“right” or “left”) with no loss to signal cohesion. Whether the “Qubit” is spun “right” or “left,” it enters the next switch gate in the circuit after the same number of rotations. Thus, because the “right” or “left” spin cancels out, a trinary quantum switch gate has 3 basic functions: ø (“closed”), 1/2 (“left” or “right”) and 1 (“open”).

Tetrahedral Tessellation Net

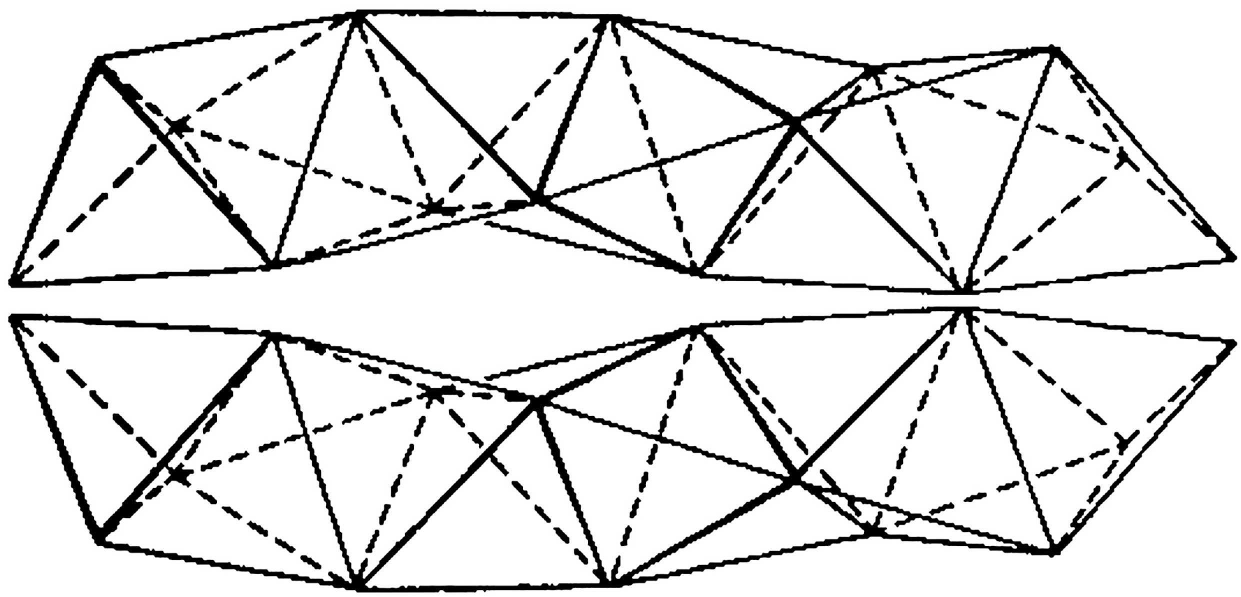


Mapping a trinary quantum switch gate onto the surface area of a tetrahedron involves taking the twin, binary, scalar opposite poles of (ψ) the figurative vector's combined coordinates of trajectory (θ) and velocity (ϕ), and translating them to the 4, triangular "faces" of the 3-simplex. Because in a trinary quantum switch gate, there are 4 possible states (open, closed, right and left), these can easily be plotted out as traits onto the faces of the tetrahedron. The twin, polar traits of velocity (ø = closed or slow and 1 = open or fast) and of trajectory (right or left as clockwise or counterclockwise, arbitrarily) maybe symbolized as opposite faces from one another, such that two flat plane-space faces sharing a common linear edge maybe labeled "open" and "closed," and the two remaining faces labeled "left" and "right." The purpose for doing this is to create a regular polytope format for expressing each of the 8 sub-quadrants on the surface of a sphere divided by 3 great arcs - 1 equator and 2 meridians. If each tetrahedron maps to a single trinary quantum switch gate, and 8 such tetrahedrons can be mapped onto the volume of a sphere, then we can see that the model of a binary "qubit" provides less encryption potential than a larger mathematical construct sphere could. In the next section, we will be considering how a Riemann sphere can be made of 4 Qubit-pairs. Such a model provides a far superior format for encryption as it multiplies the uncertainty factor. If one wishes to substitute the traditional "blockchain" model for digital crypto-currency with a sufficiently comparable trinary quantum switch gate based protocol - e.g. a model based on tetrahedral base-units instead of binary "1's and 0's" - one would only need to make a "tetra-helix" of any number (from ø to ∞) of tetrahedral trinary quantum switch gates. A tetra-helix of tetrahedra connects one tetrahedron's face to another in an essentially linear sequence, such that the faces labeled "left" and "right" (spin) are always exposed and one circuit's "open" face connects to the next circuit's "closed" face and one circuit "closed" connects to the next one "open."





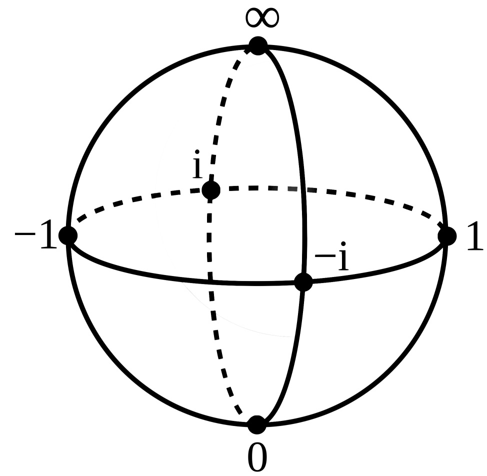
Tetrahelix Blockchain Radius



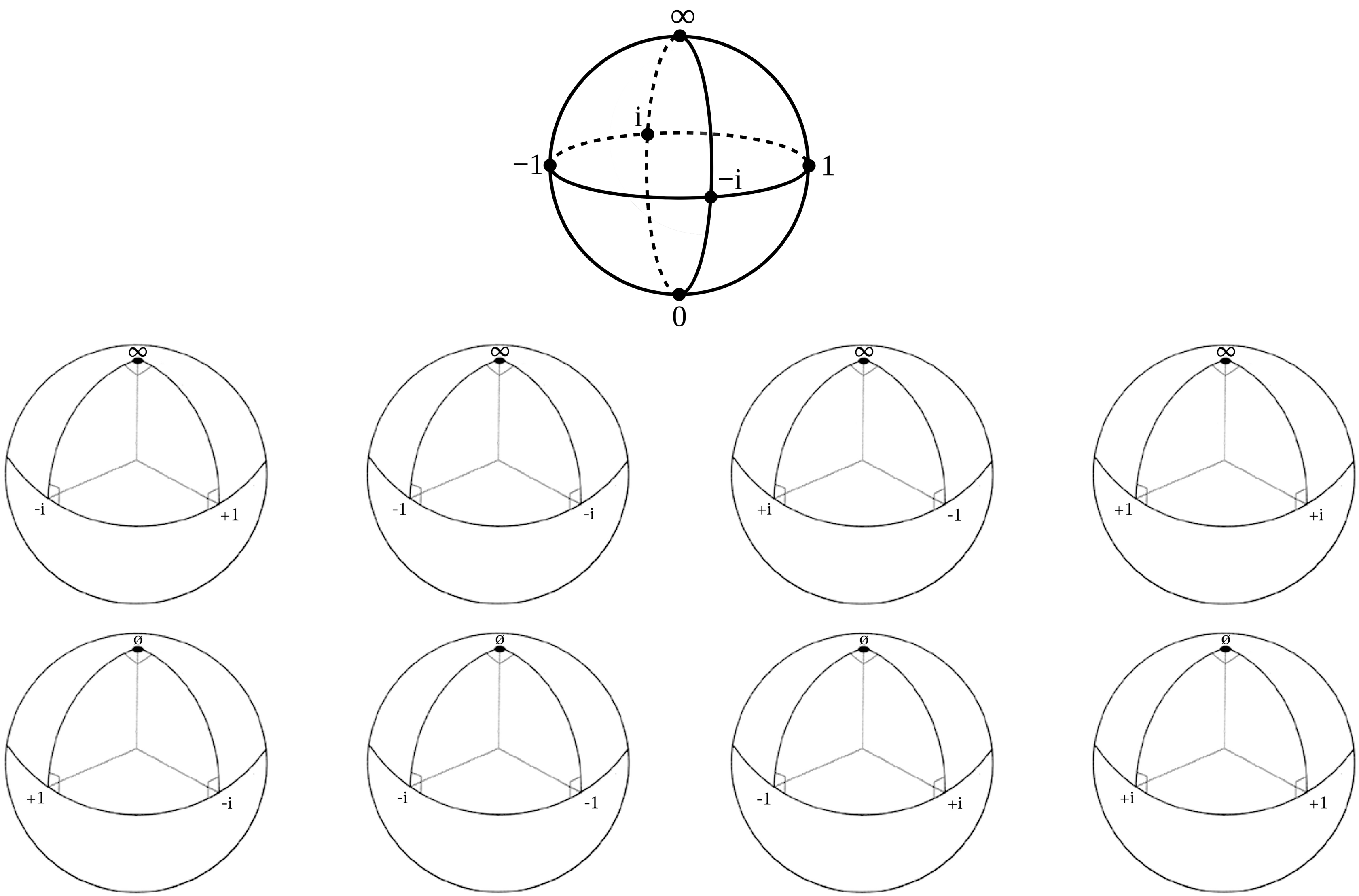
Although the faces of the individual tetrahedra labeled “left” and “right” remain “unpaired” in a tetra-helix based on the pairing of “open” and “closed” faces to form it, the “left” and “right” handedness or “chirality” factor reintroduces itself in the tetra-helix arrangement, where a rotation starting from an “open/closed” pair will gradually precess in a “clockwise” or “left-handed” spiral and one beginning from a “closed/open” merging will precess in a “counterclockwise” or “right-handed” vortex. One can deform the tetrahedra in such a tetra-helix so that they will reach a point of exact repetition of placement and angle of a tetrahedron after a certain, given length cycle, however a regular 3-simplex polytope or tetrahedron repeated in a tetra-helix pattern will never repeat itself at exactly the same angles around its central axis. In this regard the tetra-helix of regular tetrahedra mimics the “infinite decimal,” irrational nature of the transcendental number sum π. This will come into play in a subsequent section where we begin to map the blockchain radius to the spherical surface area. Because of the “chirality” of the tetra-helix, neither the “left-handed” nor “right-handed” model is fit to stand for the diameter of the Bloch sphere or “Qubit,” that axis connecting the opposite poles of |ø> and |1>. However, because of this same “chirality” factor, a tetra-helix model can be mapped onto the two radii that combine into the diameter of a “Riemann sphere.” This diameter connects ø at one pole and ∞ at the opposite pole, with (essentially) the “absolute value” of one (expressed as “|1|”) at this axis’ centroid point. So, the radius for one hemisphere of the Riemann model maybe expressed as a tetra-helix of length -1/2 between ø and |1| and the radius for the opposite hemisphere maybe expressed as a tetra-helix of length +1/2 between |1| and ∞. The value of the centroid point connecting these oppositely rotating tetra-helices being (essentially) the “absolute value” of one means these opposite, twin vortices both branch off from a single, central “genesis block” at the centroid point’s location and this block acts “achirally” - having symmetry along all its axes of rotation. The value of the tetra-helix from ø to |1| is negative (closed) and clockwise (left) and that from |1| to ∞ is positive (open) and counterclockwise (right).

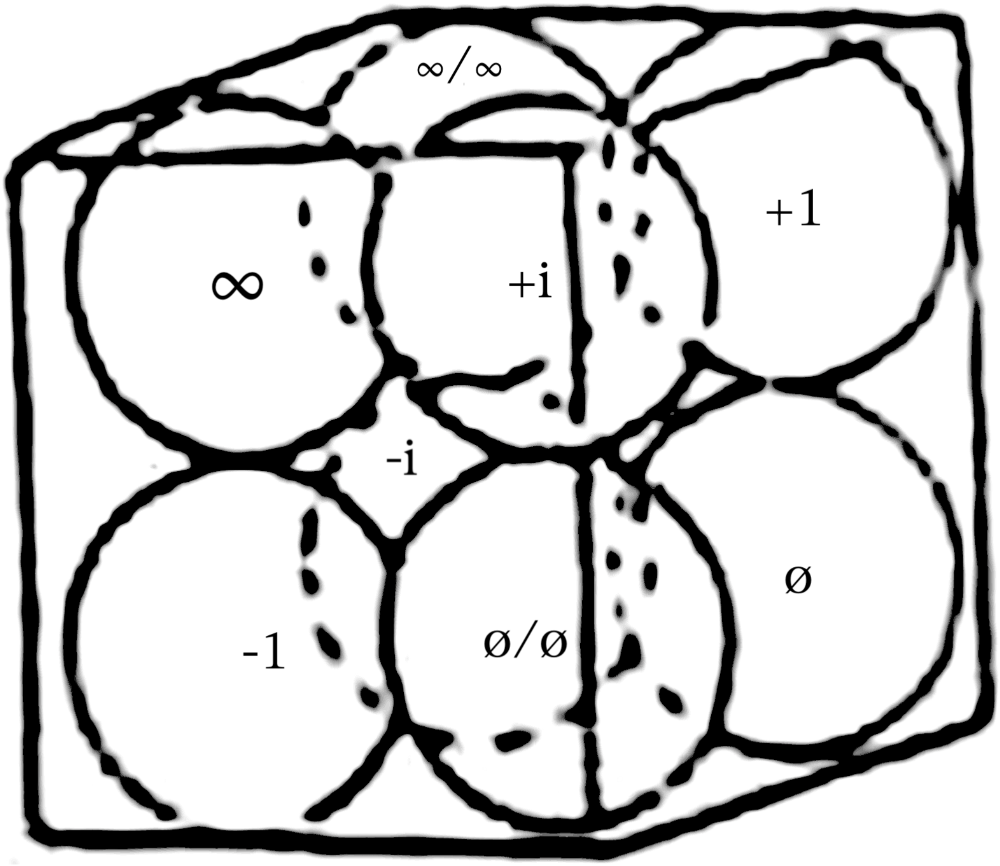
Phase 2:

8 Qubits of the Riemann Sphere

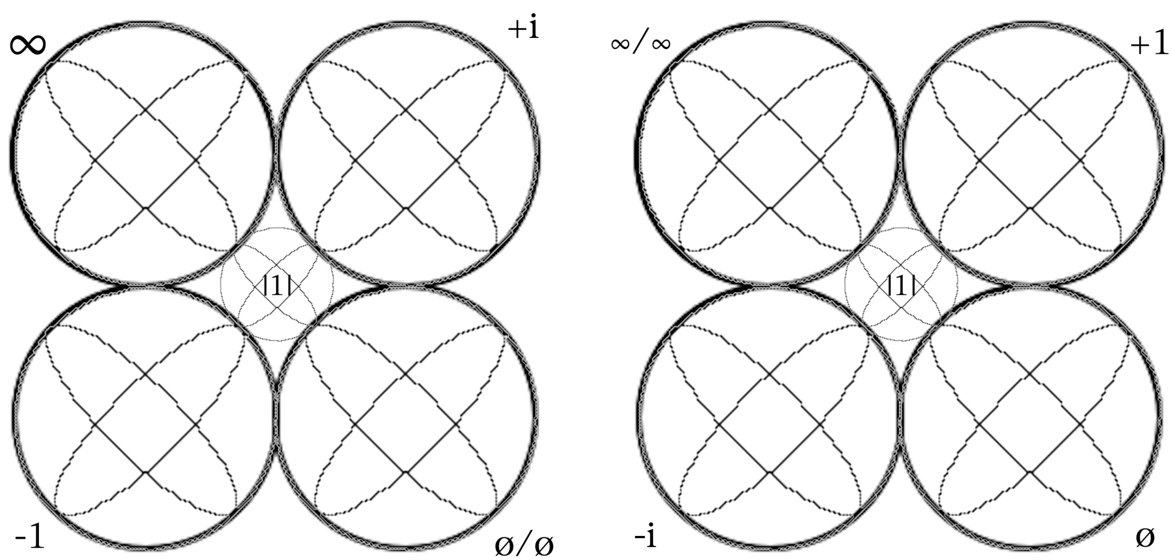


The Riemann sphere measures the "extended complex plane" (of all "complex" numbers plus ∞) as mapped onto the surface of a sphere. At one pole is "zero" (ø - the "empty set") and at the opposite pole is "infinity" (∞); at the centroid of the axis drawn between these twin poles is (essentially) the "absolute value" of one, or |1|. The reason the centroid point is labeled as "essentially" |1| is that the expanded circumference of the sphere around its equator measures opposite positions that are all derived from the |1|: first, +1 is derived from -1^2; second, -1 is derived from -√+1; third, +i is derived from +√-1; and fourth, -i is derived from -√-1. We can substitute "Qubit" Bloch spheres for each of the 8 sub-quadrants of the Riemann sphere, however only 4 of these will be "true" Qubits - those below the equator of |1| - while those above the equator are "Qubits-in-name-only." The 4 "true" Qubits are spheres with poles of ø & +1, ø & -i, ø & -1, and ø & i. The 4 "pseudo-Qubits" are spheres with poles of ∞ & +1, ∞ & -i, ∞ & -1, and ∞ & i. Because all these share a common centroid-point at |1| in the Riemann sphere, they can also each be catalogued as a Bloch sphere with poles at |1| and their respective number sum; in this case, only the sphere with poles ø and |1| is a "true" Qubit. If we measure each sub-quadrant as a Bloch sphere, then there will be 8 equal spheres packed such that they each fit into one of the 8 corners of a cube. In this model, each smaller Bloch sphere has one pole at |1| and the opposite at its respective number sum. Because there are only 6 intersection-points on the Riemann sphere's surface to divide it into the 8 equal areas, there are 2 smaller Bloch spheres in this arrangement than there are labeled intersection-points on the Riemann sphere. If one labels each sub-quadrant by one of the intersection-point values alone, one will have 2 sub-quadrants left over. In a cube of 8 smaller Bloch spheres packed into the cube's 8 corners, each symbolizing one sub-quadrant on the Riemann sphere, there will be 2 "extra" smaller Bloch spheres than there are values for labeling poles in the greater Riemann sphere. 8 sub-quadrants, each symbolized by a smaller Bloch sphere, means 2 such spheres do not have labels for their "exterior" poles - those facing away from the centroid-point of |1|. As a place-holder, it is possible to label these "extra" Bloch spheres "ø/ø" and "∞/∞", both of which values remain "undefined" in modern math. The 8 sub-quadrants on the Riemann sphere can be considered by their triple coordinates: A. The sub-quadrant of (∞, +1, -i); B. That of (∞, -i, -1); C. Of (∞, -1, +i); D. (∞, +i, +1); E. The sub-quadrant of (ø, +1, -i); F. That of (ø, -i, -1); G. Of (ø, -1, +i); and H. (Ø, +i, +1). Here, we may label sub-quadrant A. "+1;" B. "-i;" C. "-1;" D. "+i;" E. "Ø;" F. "∞;" G. "∞/∞;" and H. "Ø/Ø." The reason for transforming the 8 sub-quadrants on the surface area of a Riemann sphere into 8 smaller Bloch spheres packed into a cube is to demonstrate the premise that data, encrypted by being mapped onto a sphere, can be related - one sphere to another - in a cubic system. The Bloch sphere measures spin as vector; packing spheres into cubes and labeling them with a single number relating to their spin will be discussed in Phase 3.

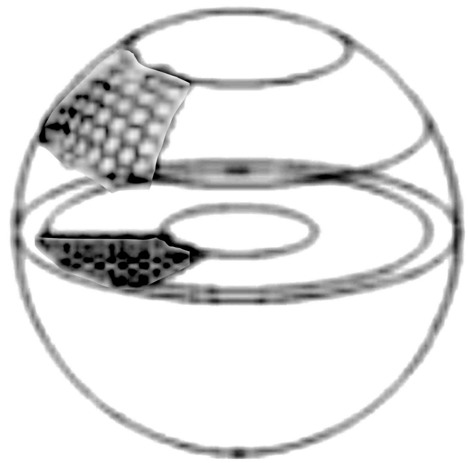

∞
i
−1
1
−i
0
∞
-i
+1
∞
-1
-i
∞
+i
-1
∞
+1
+i
ø
+1
-i
ø
-i
-1
ø
-1
+i
ø
+i
+1



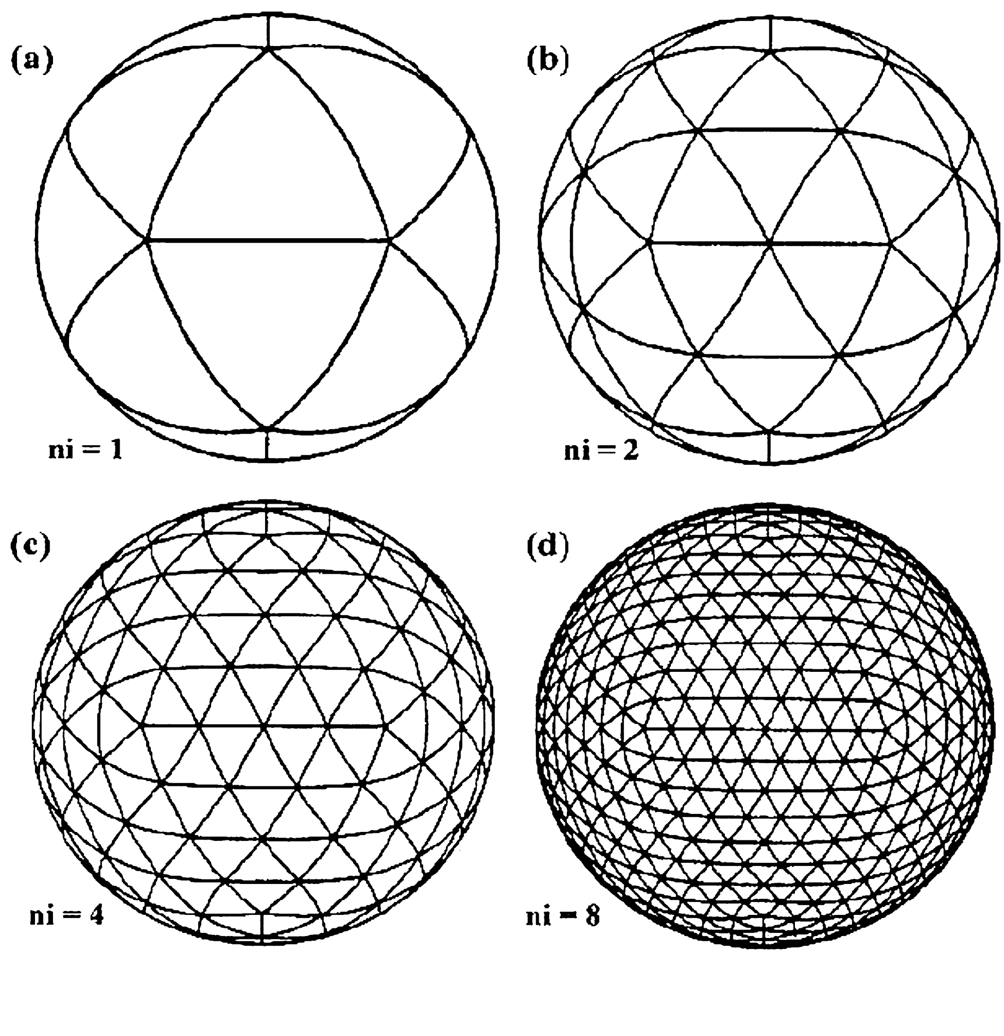
A combination of 8 “Qubits” - 1 per each sub-quadrant on a Riemann sphere’s surface - into a single cube, whose edges are all equal to the diameter of the Riemann sphere, where each “qubit” is labeled by a single scalar, binary pair of opposite traits, and where one pole = |1| and the other = any of the 6 Riemann variables. Spheres with poles labeled by opposite traits are aligned opposite one another along the $\sqrt{3}$ diagonal of the cube.

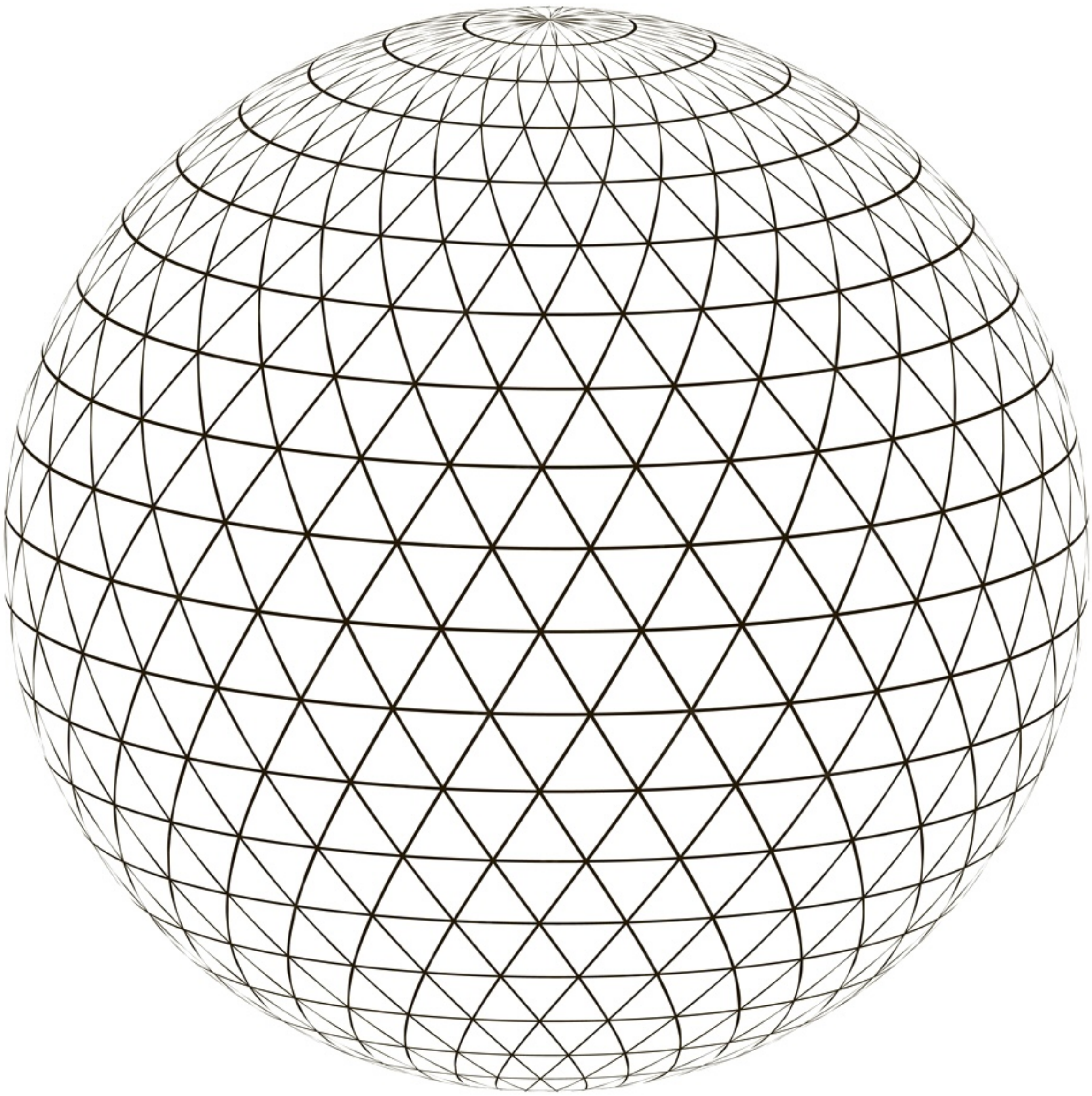


Mapping Tetrahedra Onto Spheres



The simplest 2-d polyhedron is the triangle, and so it is an ideal signifier for a “trinary” circuit. The 3-d “simplex” is the tetrahedron of 4 equal triangles, and it can function as a single “trinary” circuit. By combining a sequence of such tetrahedra into a tetra-helix, we may establish it as the radius (and/or diameter) of a sphere. Doing so allows us to map the surface area of the sphere in a sum of such “trinary” circuit signifiers, or triangles. Using a Riemann sphere in particular allows us to map the tetra-helix radius (or diameter) as potentially “infinite” or as a limitless possible supply. Using an “infinite” tetra-helix blockchain imitates the function of current cryptocurrency blockchains of having no “maximum capacity” placed on the sum of “coins” that can be “mined.” Using a “finite” tetra-helix blockchain imitates having such a “max cap” on the quantum-coin denomination. This allows for the possibility of multiple different parameters to be applied to designing multiple, different quantum-coin denominations. All such denominations would remain mappable onto a sphere as a series of N-possible triangles.





What remains now is for this “Riemann-like” sphere, with surface area mapped by N-possible triangles (where N is any complex number-sum up to and including ∞), to be able to transform the expansion of its radius (and diameter) in length into “complex-ification” of the sphere’s surface as mapped by triangles. In short, the “volume” of this figurative “sphere” must not increase, even while its apparent “radius” increases. In order to translate the tetrahedral unit-bits in the “blockchain” into the same sum of triangles on the surface of a sphere, while not expanding the circumference of this sphere, requires the “size” of the tetrahedra and the triangles on the sphere’s surface area to be variable. Thus, a “sphere” could begin with a “diameter” of 1, and so its surface area would be covered by a single “triangular” mapping; but a “sphere” adjacent to it with an area (and topological covering) measured by any complex number (N –> ∞) of triangular elements would be of the same essential dimensions in size. Each such sphere measures a different denomination of quantum crypto-currency.

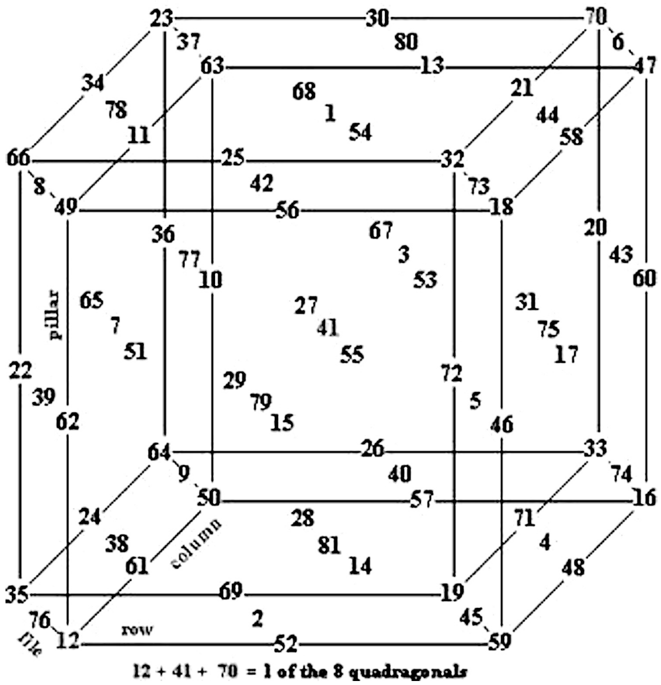
Phase 3:

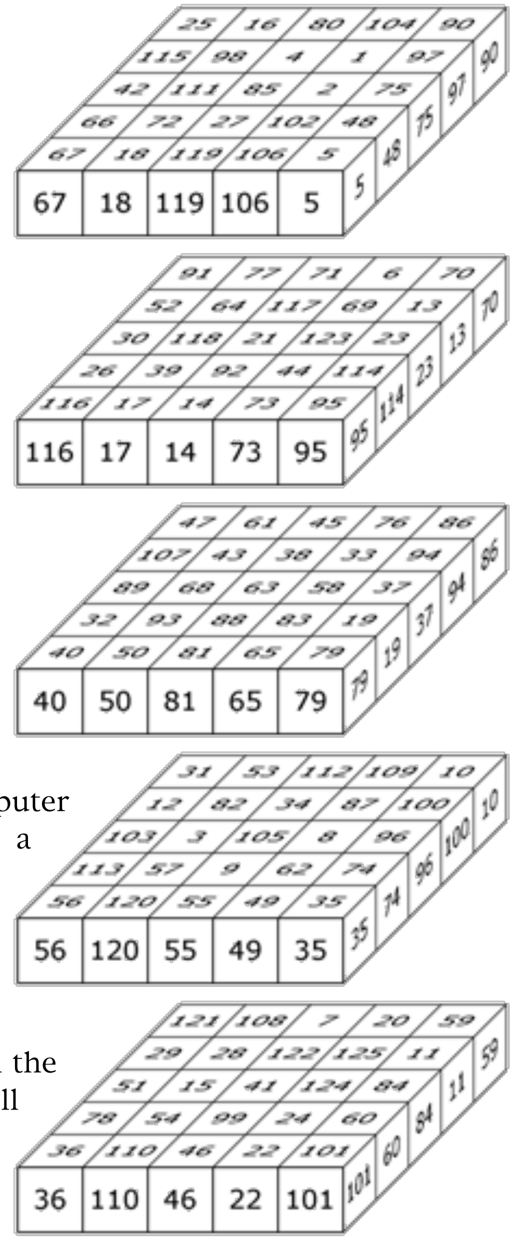
Spinors - Quantifying Spheres as Sums

Establishing a stable network at a scale where “trinary” code can exploit “uncertainty” requires quantum entanglement. Building storage for “Qubits” comprised of multiple “bits” of information - such as a single photon entangled to a sequence of “pulsed” electrons (with their poles reoriented “left” or “right” to signify “0’s and 1’s” in a binary, essentially still “Morse code”-like, signal) - requires creating a structure capable of trapping and holding such “qubits” both singularly and in multiples. While the “Bloch” and “Riemann” spheres provide layers of encryption in the programming code of these structures, the structures still have to physically exist in some specific location. An “app” that can store multiple denominations of quantum cryptocurrencies in a single figurative “wallet” can be set up online, but the actual “blockchain” as a trail of receipts for transactions has to be stored inside an actual “quantum server.” While such “quantum computers” may be far smaller, faster and literally cooler than modern electronic devices dependent on digital “server farms,” there are not yet enough of them (c. 2024) to build such a quantum cryptocurrency model. Because a “quantum internet” with “cloud storage” capable of supporting a quantum cryptocurrency model does not exist yet, there remains time to develop further layers of encryption to best secure this model. The simplest means of “stacking” encryption layers onto quantum cryptocurrencies using this method is to quantify each “spherically mapped” denomination as a single sum determined by introducing random “spin” to it - such as by striking such a “Qubit” with an external photon. This means each “sphere” model would have to be an entangled cloud of electrons (essentially like an “atom” but with no real nucleus) stored in a “quantum server,” and each “transaction” would be measured by striking this with an external photon in order to alter its “spin.” One can then sum each whole “sphere” up by the single quantity associated with this “spin” as it is measured in this way; this single “sum” would then quantify an encrypted receipt for the total number of transactions measured by the sphere. So, to “read” how many units of "quantum cryptocurrency” one has of any given denomination requires striking an electron-cloud (defined by an “uncertain” and variable sum of component electrons) with a single photon inside a quantum computer that is then linked by a server to a receiver app accessible online. So, for each such “transaction” computed in this way, another “electron” (or electron-like element) would be added to the entangled cloud that constitutes the actual “account” of the cryptocurrency in a quantum “bank.” Thus, instead of current means of digital encryption, entangled “qubits” would constitute a quantum blockchain that would be completely rewritten with each photon impact measuring a “transaction” event. The result of this for the “Qubit” blockchain (and thus for the triangularly mapped spherical surface of the “electron-like” cloud) would be somewhat similar to a player’s “move” in the game of “go,” where each added piece retiles a certain portion of the entire board, only in the case of a quantum cryptocurrency transaction, the entire figurative “board” would be re-tiled, and using trinary “Qubits” rather than binary (black or white) player pieces. Thus, any model of quantum cryptocurrency reliant on any kind of “blockchain” style apparatus to store receipts of transactions remains limited by quantum server storage space for “banking” and available internet bandwidth for transaction speed.

Perfect Magic Number Hypercubes

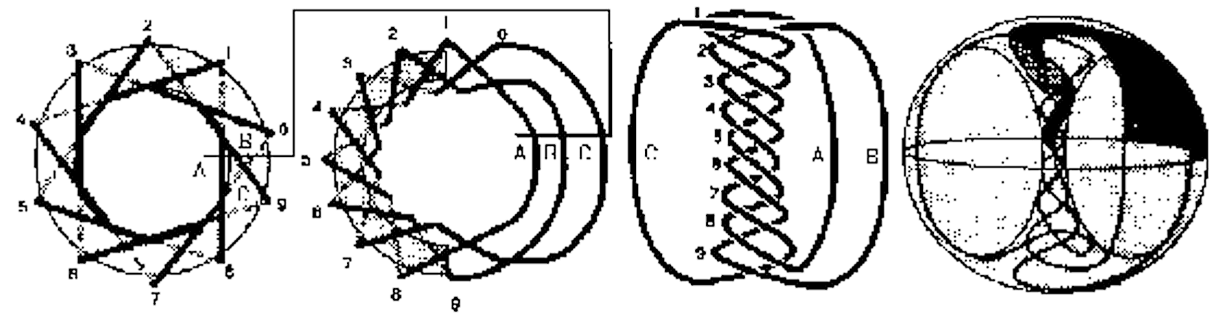
Let's say someone has a collection of different denominations of coins, and they want to add them all up and see how much they have of each, and how much they have in total. These sums amount to the "balance" of an account, that is, to how many coins one owns. So, what would be the equivalent of an account "balance" for quantum cryptocurrency? If a single unit of exchange in this economy is a "trinary" circuit, and any sum of these circuits maybe mapped onto the surface of a sphere - such that each sphere measures a different denomination of coin - then the total "balance" of one's account would be equal to the sum of all "trinary" circuits multiplied by the sum of all spherical denominations. One could thus have - for example - 5 of one denomination of coin, and 10 of another, and relative to their individual "value" (sum of trinary qubits) one could then account for the total balance of their account (15 coins, 5 of one value, 10 of another). What would be the physical equivalent of this in a quantum computer storing and filtering access to a quantum cryptocurrency account? Assigning each "sphere" (or denomination) a unique complex number (such as by striking it with an exterior photon) we derive a label for each that entangles the number of "electron-like" elements (trinary circuits or individual units of value) with the number signified by ψ - the "vector" sum combining the "frequency" (v) and "wavelength" (λ) as θ and ϕ of the whole entangled "electron-like" cloud overall. This single complex number assigned to each such "sphere" or denomination per such "transaction" event is categorized as one such "spherical" sum in a lattice of indeterminate capacity, containing limitless other such "spheres" - each assigned its own singular, complex number sum. Each "sphere" in this "cubic" lattice therefore has its own unique "spin" or "vector" as an "access code" and these are sustained in a quantum superposition of "uncertainty" except during a "transaction event" accessing them - in this way, the "spin" or "vector" complex number sum cannot be read without conducting a "transaction" by striking a photon against that particular, "electron-like" sphere. This is the process by which "mining" new "crypto-coins" occurs: each time the "spherical account" is accessed, another trinary circuit is added to it. When the spheres are not being accessed, they remain in a condition of "uncertainty" or a "locked cipher," but during the transaction event, they collapse into a measurable probability that can be assigned a single complex number sum and their encryption "un-locks." When the "electron-like" spheres are accessed during a transaction event by being struck by a photon, they become a definite complex number sum - the "hash code" for each unique transaction - and these can best be "balanced" (while also being arranged asymmetrically for increased cryptographic complexity) when the lattice is most alike a "perfect magic number cube" (or tesseract in any higher dimension). In a "perfect magic number cube" the number sums in the cells of all the vertical pillars and columns, all the horizontal rows, the "space-diagonals" and all the diagonals (for each orthogonal slice) add up to the same "magic number constant." This number can then be used to describe the sum of the entire lattice system, however only temporarily, while the transaction event is taking place; the rest of the time, these numbers are scrambled in "uncertain" quantum super-position. Such is the quantum computing "bank account" holding all the different denominations of quantum cryptocurrencies accessible by an online app.



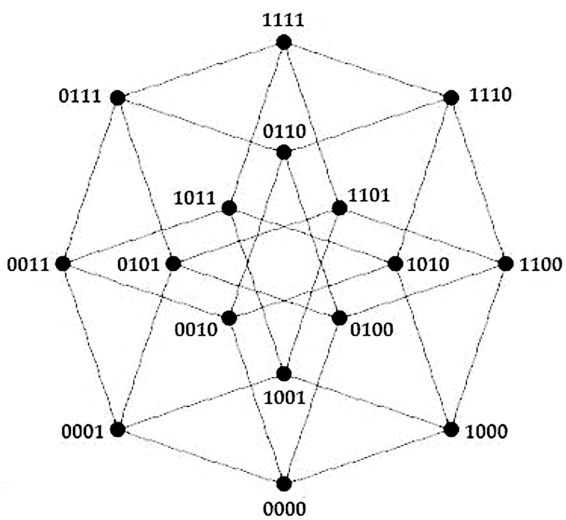


Each quantum cryptocurrency "bank account" would be stored, either in a single quantum computer server or else distributed in a "cloud" across a quantum internet. Each such "account" would be comprised of one or more denomination spheres - or "electron-like" clouds storing multiple trinary circuits in an "uncertain" condition of super-position. These "electron-like" spheres would resemble atoms of various chemical elements, given the sum of electrons that can occupy each orbital shell in natural chemistry, however these spheres stored in the quantum computing server would not have core nuclei, and would only be entangled in the context of this atomic-scale lattice. Each such "bank account" or atomic-scale lattice could be stored as a cubic system and its component elements arranged relative to one another alike the number sums inside cells in a standard "magic number" square, cube or hypercube. This system would be sufficiently randomized when the individual cell's sums are not being directly measured (as in a transaction event), however whenever one cell's number sum could be ascertained it would increase the probability of predicting the number sums in the neighboring cells. Therefore, these numbers are all constrained in this "atomic lattice" inside a quantum computer into a "perfect magic number hypercube" that is always balanced but also always scrambled, such that the same ratio for the "magic number constant" will always apply, but such that the numbers will always change, so no single "magic constant" can be predicted without a "transaction event" measuring the system.

Further Implications



The Galois Tesseract



The 16 subsets of a 4-set or the 16 points in the affine 4-space over the two-element field

0000	0100	1100	1000
0001	0101	1101	1001
0011	0111	1111	1011
0010	0110	1110	1010

The same 16 subsets or points can be arranged in a 4×4 array that has, when the array's opposite edges are joined together, the same adjacencies as those of the above tesseract.

Thus the 4×4 array is also a tesseract, which, because of its usefulness in picturing Galois geometry, we may call *the Galois tesseract*.

Further insights into mapping strings (such as block-chains) onto (4-d spherical) torus shapes maybe found in the complex, modern QBLH works of Stan Tenen for the Meru Foundation. Applications involving mapping magic number squares onto higher dimensional forms - such as the Galois model that uses the 16 vertex corners of a tesseract to symbolize the 16 number sums in a standard 4X4 magic number square - also remain open for interpretation in the contexts of quantum cryptography generally and of quantum cryptocurrency specifically. The basic premise of such a project is that the more levels of encryption, the better the privacy for the end-user. While adding extra steps to encryption processing used to take a commensurate extra sum of time, in quantum computing this time-lag becomes relativistically infinitesimal. Therefore, the more levels of encryption and the more random the scrambling methods can be made, the better. Thus, ultimately, quantum cryptocurrencies will begin to be designed with all different kinds of encryption models - some possibly similar to those depicted here, though most likely very different. The models and ideas for such a quantum cryptocurrency project are here presented as a novel mental exercise, meant only to inspire further research into the topic of emergent economic currencies.

Citations

• “Gaussian Source Coding With Spherical Codes,” Jon Hamkins, Member, IEEE, and Kenneth Zeger, Fellow, IEEE; IEEE TRANSACTIONS ON INFORMATION THEORY, VOL. 48, NO. 11, NOVEMBER 2002.

• “Finding and Investigating Exact Spherical Codes,” Jeffrey Wang; November 10, 2018. https://arxiv.org/abs/0805.0776v2

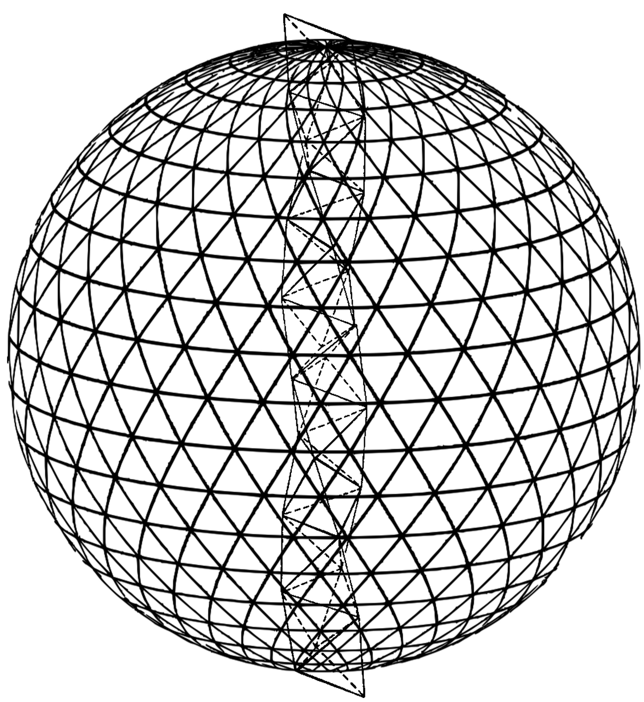
• “Sphere Packings, Lattices and Groups. Material for Third Edition” J. H. Conway and N. J. A. Sloane; Sept. 16, 1998.

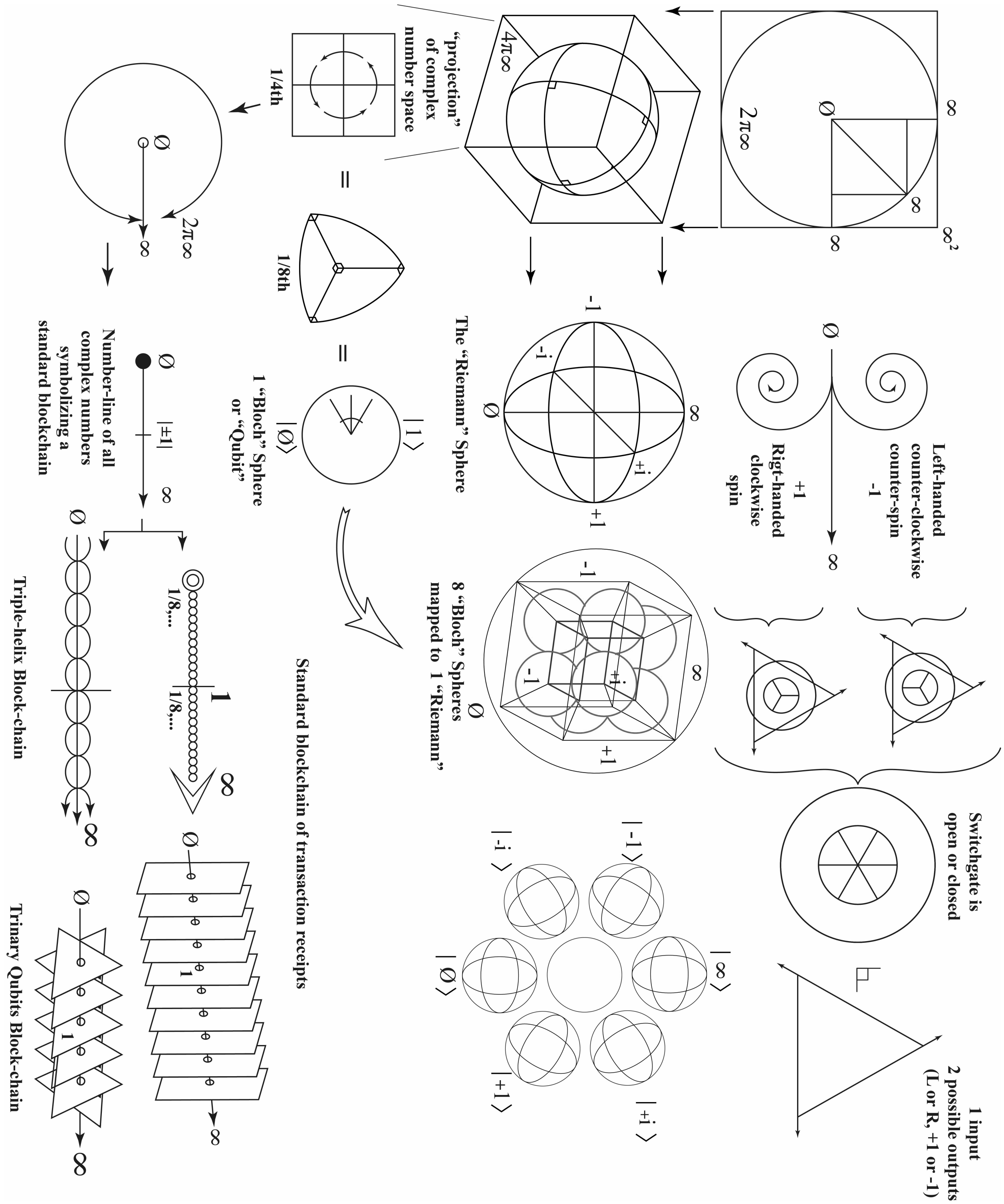
• “SPHERE COVERINGS, LATTICES, AND TILINGS (in Low Dimensions),” FRANK VALLENTIN; Technische Universita¨t Mu¨nchen, Zentrum Mathematik. Die Dissertation wurde am 18. Juni 2003 bei der Technischen Universita¨t Mu¨nchen eingereicht und durch die Fakulta¨t fu¨r Mathematik am 26. November 2003 angenommen.

• “The Sphere Covering Inequality and Its Applications,” Changfeng Gui and Amir Moradifam; October 28, 2016. https://arxiv.org/abs/1605.06481v3

•“An algorithm for making magic cubes,” Marian Trenkler; Catholic University, A.Hlinku 56, 034 01 Ruˇzomberok, Slovakia; The ΠME Journal, Vol. 12, No. 2, pp. 105-106, Spring 2005.

• “The Zen of Magic Squares, Circles, and Stars: An Exhibition of Surprising Structures,” Pickover, Clifford. (2001).





Original notes by: Rev Jonathan Barlow Gee, July 24, 2024.